How we teach calculations:

Trinity First School Calculation Policy for Mathematics

May 2014

TLC

About our Calculation Policy

The following calculation policy has been devised and agreed by teachers at Trinity First School to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, Please note that early learning in number and calculation in Foundation Stage follows the 'Development Matters' EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age stage expectations

The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014, **however at Trinity we believe it is vital that pupils are taught according to the stage that they are currently working at,** being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This is an essential component of our calculation lessons.

Choosing a calculation method:

Children will be taught and encouraged to use the following processes in deciding what approach they will take to solve a calculation, and to ensure they select the most appropriate method for the numbers involved:

Can I do it in my head using a mental strategy?

Could I use some jottings to help me?

Should I use a written method to work it out?

To work out a tricky calculation:	
R	read
U	understand
C	calculate
S	solve
A	answer
C	check

Addition
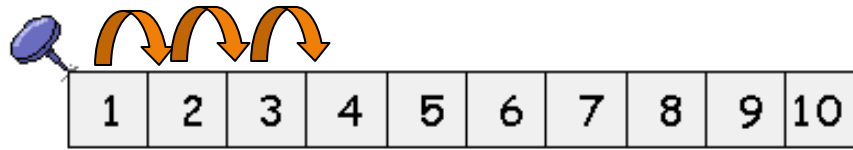
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines and be shown numbers in different contexts.

- In practical activities and discussion, begin to use the vocabulary involved in adding.

- Find the total number of items in two groups by counting all of them

- Say and use number names in order in familiar contexts

- Recognise one more than a number.

- Begin to read and write the addition (+) and equals (=) signs within number sentences.

3 + 7

4 + 3 = 7
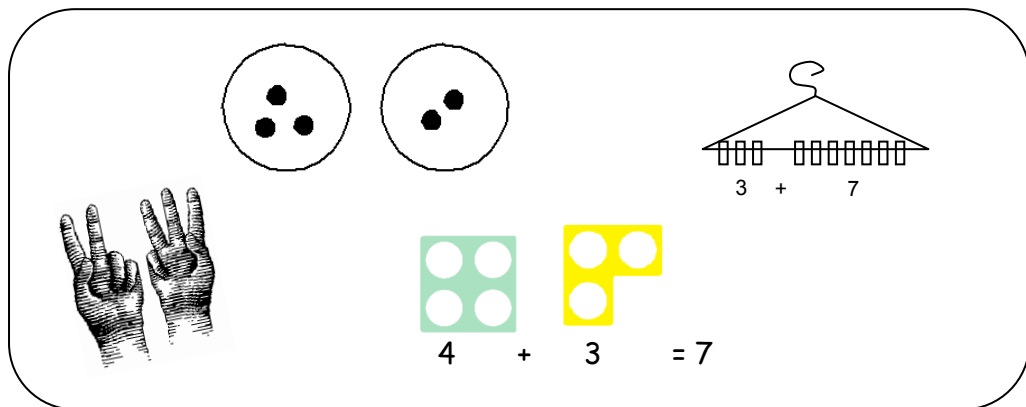
Key vocabulary: *add, more, and, make, altogether, is the same as , double, most, count on, number line, number track*

Key skills for addition at EYFS

- Count objects by saying one number name for each item
- Begin to represent numbers, using fingers, marks on paper or pictures
- Recognise numerals at least 1 to 10
- Count forwards within the number sequence
- Count actions or objects that cannot be moved
- Select the correct numeral to represent the number of objects in a set
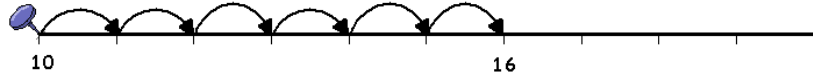- Match and compare number of objects in two sets

Addition

Year 1 Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on.

10 + 6

Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines (including blank number lines), and be shown numbers in different contexts.

- Read and write the addition (+) and equals (=) signs within number sentences, understanding the = sign as 'is the same as' and can be in any place.

- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: 8 + 3 = □   15 + □ = 19
5 + 3 + 1 = □   6 = □ + □

- This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

Key vocabulary: *add, more, and, make, altogether, double, most, count on, number line, number track, is the same as,* equal to, total, equals, plus, addition

**Key skills for addition at Y1:**

- Read and write numbers to 100 in numerals, incl. 1—20 in words
- Recall bonds to 10 and 20, and addition facts <u>within</u> 20
- Count to and across 100
- Count in multiples of 1, 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations
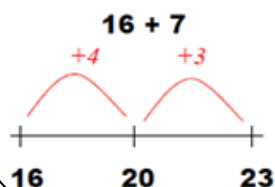- Understand the place value of 2-digit numbers (tens and ones)

Addition

Year 2 Add with 2-digit numbers
Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.
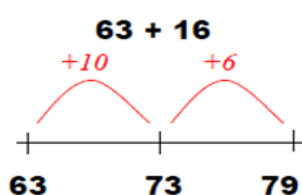
Add 2-digit numbers and tens:

27 + 30
+10 +10 +10

27   37   47   57
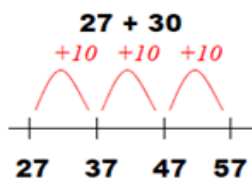
Add 2-digit numbers and units:

16 + 7
+4   +3

16   20   23

Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:

63 + 16
+10   +6

63   73   79

23 + 34:

```
   2 0 + 3
 + 3 0 + 4
   5 0 + 7
       = 5 7
```

STEP 1: Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).

58 + 43:

```
 5 0 + 8
 4 0 + 3
 9 0 +11
     =1 0 1
```

STEP 3: Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers (see Y3).

To support understanding, pupils may physically make and carry out the calculation with Dienes Base 10 apparatus or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.

10 + 8 = 18

Key vocabulary: *add, more, plus, and, make, altogether, total, is the same as, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary*

Key skills for addition at Y2:
- Add a 2-digit number and ones (e.g. 27 + 6)
- Add a 2-digit number and tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 47)
- Add three single-digit numbers (e.g. 5 + 9 + 7)
- Show that adding can be done in any order (the commutative law)
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number
- Understand the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using < > and = signs
- Read and write numbers to at least 100 in numerals and words

Addition

Year 3 Add numbers with up to 3-digits

3	0	0	+	2	0	+	4
2	0	0	+	7	0	+	2
5	0	0	+	9	0	+	6
				=	5	9	6

Introduce the expanded column addition method:

> Add the **units** first, in preparation for the compact method.

Move to the compact **column addition** method, with 'carrying':

In order to carry out this method of addition:

- Children also need to recognise the value of the hundreds, tens and units without recording the partitioning.

- Pupils need to be able to add in columns.

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to 'carrying' for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

> Add **units** first.

> 'Carry' numbers **underneath** the bottom line.

```
  236
+  73
─────
  309
 1
```

> Remind pupils the actual value is '**thirty add seventy**', but we say '**three tens add seven tens**' because it is in the tens column.

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary*, **hundreds boundary, increase, vertical, 'carry', expanded, compact**

Key skills for addition at Y3:
- Count, read and write numbers to 1000 in numerals and words
- Add 2-digit numbers mentally, incl. those exceeding 100
- **Add a three-digit number and ones mentally (175 + 8)**
- **Add a three-digit number and tens mentally (249 + 50)**
- **Add a three-digit number and hundreds mentally (381 + 400)**
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 1000 and adjusting, using near doubles, partitioning and recombining

Addition

Year 4 Add numbers with up to 4 digits

Move from expanded addition to the compact column method, **adding units first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913

Add **units** first.

Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

'Carry' numbers **underneath** the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **not 5 add 3**, for example.

Use and apply this method to money and measurement values.

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact,* **thousands, hundreds, digits, inverse**

Key skills for addition at Y4:

- Select most appropriate method: mental, jottings or written and explain why
- Recognise the place value of each digit in a four-digit number
- Round any number to the nearest 10, 100 or 1000
- Estimate and use inverse operations to check answers
- Solve 2-step problems in context, deciding which operations and methods to use and why
- Find 1000 more or less than a given number
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods to use and why
- Estimate and use inverse operations to check answers to a calculation

Addition

Children exceeding Y4 expectations will undertake work including money, measures and decimals with different numbers of decimal places.

£ 2 3 . 5 9
+ £ 7 . 5 5
£ 3 1 . 1 4

The decimal point should be aligned in the same way as the other place value columns, and must remain in the same column in the answer row.

2 3 4 8 1
+ 1 3 6 2
2 4 8 4 3

Numbers should exceed 4 digits.

1 9 . 0 1
3 . 6 5
+ 0 . 7 0
2 3 . 3 6

Pupils should be able to add **more than two values**, carefully aligning place value columns.

Say '6 tenths add 7 tenths' to reinforce place value.

Empty decimal places can be filled with a place holder to show the place value in each column.

Children should:

- Understand the place value of **tenths and hundredths** and use this to align numbers with different numbers of decimal places.

Key vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse* & **decimal places, decimal point, tenths, hundredths, thousandths**

Key skills for addition at Y4+

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 1000 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds
- Use rounding to check answers and accuracy
- Solve multi-step problems in contexts, deciding which operations and methods to use and why
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit
- Round any number up to 1, 000, 000 to the nearest 10, 100, 1000, 10,000 and 100,000
- Add numbers with more than 4 digits using formal written method of columnar addition
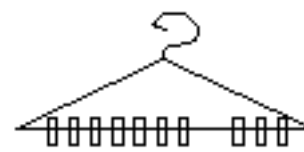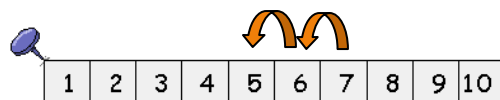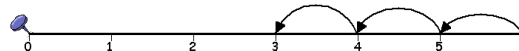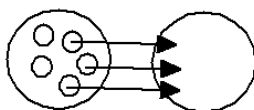
Subtraction

Children should::

- Children develop an understanding of subtraction practically, showing subtraction on bead strings, using cubes, everyday objects etc. and in familiar contexts, and are introduced to more formal recording using number lines

- In practical activities and discussion, begin to use the vocabulary involved in subtraction.

- Subtract by combining taking away (5 cubes and 3 cubes)

- Say and use number names in reverse order in familiar contexts

- Recognise one less than a number.

- Begin to read and write the subtraction (-) and equals (=) signs within number sentences

hide 3 holes

7 jump back 2 is 5

10 - 3

Key vocabulary: *take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?*

Key skills for subtraction at EYFS:

- Given a number, say **one more or one less**
- Count to and over 100, **forward and back**, from any number
- Represent and use **subtraction facts to 20 and within 20**
- Subtract with **one-digit and two-digit** numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems
- Read and write numbers from 0 to 20 in numerals and words
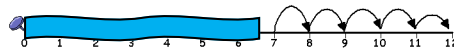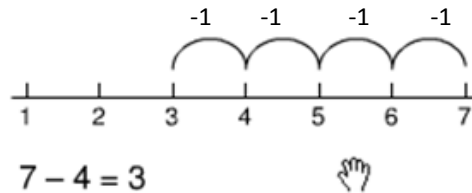
Subtraction

<u>Year 1</u> Subtract from numbers <u>up to 20</u>

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

*Read, write and interpret number sentences with - and = signs.*

Subtract by taking away

Count back in ones on a numbered number line (and a blank number line) to take away, with numbers up to 20:

-1 -1 -1 -1

$7 - 4 = 3$

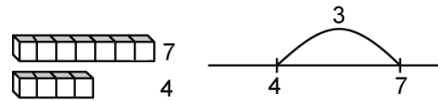*Model subtraction using hundred squares and numbered number lines/tracks and practically.*

Find the 'difference between'

This will be introduced **practically** with the language **'find the difference between'** and **'how many more to get to..?'** in a range of familiar contexts.
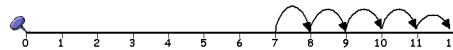
10   8
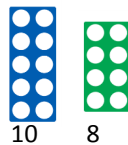
difference of 2

'Seven is 3 more than four'

7

4

3

4   7

The difference between 7 and 4 is 3.

Mental subtraction

Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero.

Key vocabulary: *take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?, difference between, equal to*

Key skills for subtraction at Y1:

- Given a number, say **one more or one less**
- Count to and over 100, **forward and back**, from any number
- Represent and use **subtraction facts to 20 and within 20**
- Subtract with **one-digit and two-digit** numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems
- Read and write numbers from 0 to 20 in numerals and words

Year 2 Subtract with 2-digit numbers

**Subtract on a number line by** <u>counting back</u>, aiming to develop mental subtraction skills.

This strategy will be used for:

* **2-digit numbers subtract units** (by taking away / counting back) e.g. 36—7
* **2-digit numbers subtract tens** (by taking away / counting back) e.g. 48—30
* **Subtracting pairs of 2-digit numbers** (see below:)

Use Dienes blocks for subtraction calculations too.

Subtracting pairs of 2-digit numbers on a number line:

47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:

Move towards more efficient jumps back, as below:

-1   -1   -1   - 10   - 10
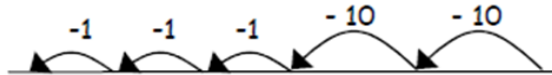
24   25   26   27   37   47

Subtract tens first.

Then subtract units.

-3   -20

24   27   47

Combine methods with use of a hundred square to reinforce understanding of number value and order.

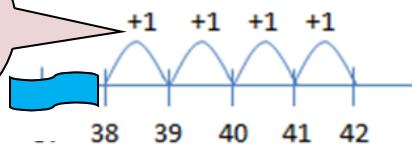Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:

-3   -2   -20
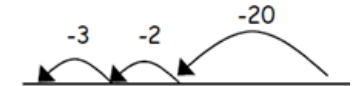
17   20   22   42

Mental strategy - subtract numbers close together by **counting on:**

42 − 38 = 4

Start with the smaller number and count on to the largest.

+1   +1   +1   +1

38   39   40   41   42

<u>Counting on</u> also needs to be taught as a key mental strategy for subtraction where children recognise numbers that are **close together** e.g. 42-38. Jottings by counting up on a number line.

Key vocabulary: *take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?* **difference, count on, strategy, partition, tens, units, equal to, difference between,**

Key skills for subtraction at Y2:

* Recognise the place value of each digit in a two-digit number.
* Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
* Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
* Show that subtraction of one number from another cannot be done in any order.
* Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
* Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
* Read and write numbers to at least 100 in numerals and in words.

Subtraction

Subtraction

Year 3 Subtracting with 2 and 3-digit numbers.

Introduce **partitioned column subtraction** method.

STEP 1: introduce this method with examples where **no exchanging** is required.

$$89 - 35 = \underline{54}$$

```
   80   9
-  30   5
   50   4
```

When learning to 'exchange', explore 'partitioning in different ways' so that pupils understand that when you exchange, the **VALUE** is the same ie **72** = 70+2 = 60+12 = 50+22 etc. Emphasise that the **value hasn't changed**, we have just partitioned it in a different way.

STEP 2: introduce 'exchanging' through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

72 - 47

```
   ⁶⁰7̶0  ¹2
-  40   7
   20   5  = 25
```

Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

STEP 3: Once pupils are secure with the understanding of 'exchanging', they can use the partitioned column method to subtract any 2 and 3-digit numbers.

```
2  3  8  -  1  4  6  =  9  2

         ¹ ⁰   ⁰
2̶ 0̶ 0 + '3 0 + 8
- 1 0 0 + 4 0 + 6
    0 + 9 0 + 2
```

Subtracting money: partition into e.g. £1 + 30p + 8p

Counting on as a mental strategy for subtraction:

Because counting on in tens is the way we use a 100 square.

```
   +10      +3
89      99  102
```

Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121—118), and also for numbers that are 'nearly' multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 131—79, or calculating change from £1 etc.).

- Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:
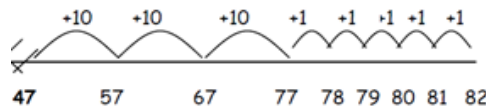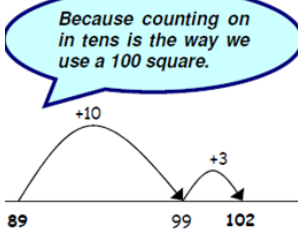
```
  +10  +10  +10  +1 +1 +1 +1 +1
47     57    67    77 78 79 80 81 82
```

Key vocabulary: *equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units* **exchange, decrease, hundreds, value, digit**

Key skills for subtraction at Y3:

- Subtract mentally a: **3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds** .
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number .
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

Approximate. Calculate. Check .
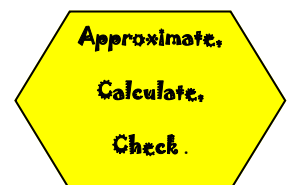
Subtraction

Year 4 Subtract with up to 4-digit numbers

Partitioned column subtraction with 'exchanging' (decomposition):

$$2754 - 1562 = 1192$$

	2000	+	700 ⟍ 600	+	50	+	4
-	1000	+	500	+	60	+	2
	1000	+	100	+	90	+	2

As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** to

Subtracting money: partition into £1 + 30 + 5 for

Compact column subtraction

$$\begin{array}{r} 2\,\overset{6}{\cancel{7}}\,{}^{1}5\,4 \\ -\ 1\,5\,6\,2 \\ \hline 1\,1\,9\,2 \end{array}$$

To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it.

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved— mental, counting on, counting back or written method.

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

Approximate, Calculate, Check.

Key vocabulary: *equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse*

Key skills for subtraction at Y4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

Subtraction

Year 4+ Subtract with at least 4-digit numbers

Children exceeding Y4 expectations will undertake work including money, measures and decimals with different numbers of decimal places.

Compact column subtraction

(with 'exchanging').

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

Subtracting with larger integers.

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Create lots of opportunities for subtracting and finding differences with money and measures.

Add a place holder in any empty decimal places to aid understanding of what to subtract in that

Approximate. Calculate. Check.

Key vocabulary: *equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, **tenths, hundredths, decimal point, decimal, integer**.*

Key skills for subtraction at Y4+

- Subtract numbers mentally with increasingly large numbers .
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy .
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through zero.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10,000 and 100,000.

Multiplication

Through grouping of small quantities, pupils begin to understand multiplication and doubling of numbers

- In practical activities and discussion, begin to use the vocabulary involved in multiplication.
- Present practical problem solving activities involving counting repeated groups of the same size
- Count Give children experience of counting in 2s, 5s and 10s.
- Begin to recognise simple doubles

How many legs will 3 teddies have?

2 + 2 + 2 = 6

Key vocabulary: *groups of, lots of, times, altogether, count, double, twos, fives, tens*

Key skills for multiplication at EYFS

- Recognise sets or groups with the same value
- Count in multiples of 2, 5 and 10.
- Begin to understand doubling using concrete objects and pictorial representations.
- They make connections between arrays, number patterns, and counting in twos, fives and tens
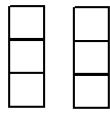
Multiplication

Year 1 Multiply with concrete objects, arrays and pictorial representations.

Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities;

- Children will have experience of counting equal group of objects in 2s, 5s and 10s.

- Present practical problem solving activities involving counting equal sets or groups, as above.
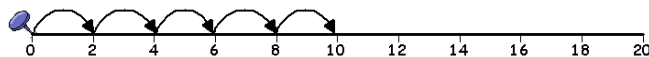
- Double sets of objects

double

Begin to use repeated addition and arrays

5 + 5 + 5

Find patterns on 100 squares

Key vocabulary: *groups of, lots of, times, altogether, count, double, twos, fives, tens array multiply, multiple of*

Key skills for multiplication at Y1:

- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.
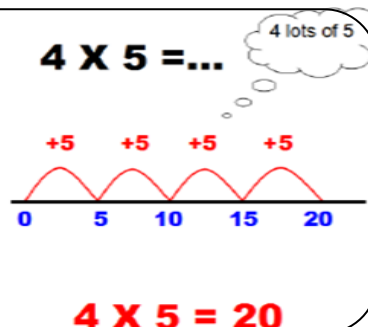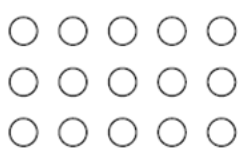
Multiplication

Use repeated addition on a number line:

- Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.
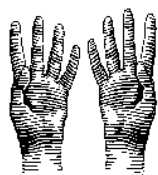
4 X 5 =...

4 lots of 5

+5 +5 +5 +5

0   5   10   15   20

4 X 5 = 20

Use arrays:

Doubling fact to 10 /20

$5 \times 3 = 15$

$3 \times 5 = 15$

$5 \times 3 = 3 + 3 + 3 + 3 = \underline{15}$

$3 \times 5 = 5 + 5 + 5 = \underline{15}$

Use arrays to help teach children to understand the commutative law of

$3 \times \square = 6$

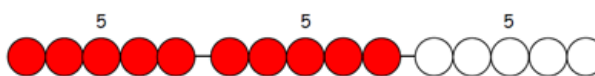$\square \times 2 = 6$

X	1	2	3	4	5
1					
2					
3					
4					
5					

Use of multiplication grids to 5x5

Use practical apparatus:

$5 \times 3 = 5 + 5 + 5$

5   5   5

Use mental recall:

- Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practice in counting and understanding of the operation.

Key vocabulary: *groups of, lots of, times, altogether, count, double, twos, fives, tens array multiply, multiple of, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...*

Key skills for multiplication at Y2:
- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements **using the x and = signs**.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

Multiplication

Year 3 Multiply 2-digits by a single digit number

Introduce the grid method for multiplying 2-digit by single-digits:

Eg.   23 x 8 = 184

X	20	3
8	160	24

| x	8
20	160
3	24

160 + 24 = 184

Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format.

To do this, children must be able to:

- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:
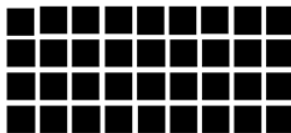
9 x 4 = 36
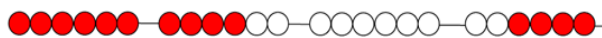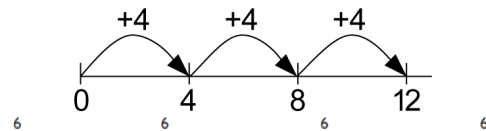
+4   +4   +4

0   4   8   12

6   6   6   6

Key vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times...,* **partition, grid method, multiple, product, tens, units, value**

Key skills for multiplication: at Y3

- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including **2-digit x single digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g using commutativity (**4 × 12 × 5 =** 4 × 5 × 12 = **20 × 12** = 240) and for missing number problems ☐ x 5 = 20,   3 x ☐ = 18,   ☐ x ☐ = 32

Multiplication

Year 4 Multiply 2 and 3-digits by a single digit, using all multiplication tables up to 12 x 12

Developing the grid method:

Eg. 136 x 5 = 680

X	100	30	6
5	500	150	30

$$\begin{array}{r} 500 \\ 150 \\ +\ 30 \\ \hline 680 \end{array}$$

Encourage column addition to add accurately.

*Move onto **short multiplication** if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, **and** are already confident in 'carrying' for written addition. (see Y4+)*

Children should be able to:

- **Approximate before they calculate**, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g:

  "**346 x 9** is approximately 350 x 10 = **3500**"

  Record an approximation to check the final answer against.

- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.

- Recall all times tables **up to 12 x 12**

Approximate, Calculate, Check.

Key vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of,* ***inverse***

Key skills for multiplication at Y4:
- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12.**
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutativity and other strategies mentally 3 x 6 = 6 x 3 , 2 x 6 x 5 = 10 x 6 , 39x7 = 30 x 7 + 9 x 7.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and units)

Multiplication

Year 4+ Multiply up to 4-digits by 1 or 2 digits.

Introducing column multiplication

- Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method.
- Children need to be taught to approximate first, e.g. for **72 x 38, they will use rounding: 72 x 38** is approximately 70 x 40 = **2800**, and use the approximation to check the reasonableness of their answer against.

Short multiplication for multiplying by a single digit

x	300	20	7
4	1200	80	28

$$\begin{array}{ccc} & 3 & 2 & 7 \\ \times & & & 4 \\ \hline 1 & 3 & 0 & 8 \\ & & _1 & _2 \end{array}$$

Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits

	10	8
10	100	80
3	30	24

$$\begin{array}{ccc} & 1 & 8 \\ \times & 1 & 3 \\ \hline & 5 & 4 \\ 1 & 8 & 0 \\ \hline 2 & 3 & 4 \end{array}$$

18 x 3 on the 1st row

(8 x 3 = 24, carrying the 2 for twenty, then '1' x 3).

18 x 10 on the 2nd row. Put a zero in units first, then say 8 x 1, and 1 x 1.

Moving towards more complex numbers:

The grid could be used to introduce long multiplication, as the relationship can be seen in the answers in each row.

$$\begin{array}{cccc} & 1 & 2 & 3 & 4 \\ \times & & & 1 & 6 \\ \hline & 7 & 4 & 0 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ \hline 1 & 9 & 7 & 4 & 4 \end{array}$$

(1234 x 6)

(1234 x 10)

$$\begin{array}{cccc} & 3 & 6 & 5 & 2 \\ \times & & & & 8 \\ \hline 2 & 9 & 2 & 1 & 6 \\ & & _5 & _4 & _1 \end{array}$$

Approximate,
Calculate,
Check .

Key vocabulary *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse,* **square, factor, integer, decimal, short/long multiplication, 'carry'**

Key skills for multiplication at Y4+:
- Identify multiples and factors, using knowledge of **multiplication tables to 12x12.**
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.
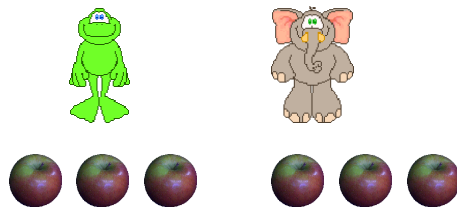
Division

EYFS Develop a sense of number within the context of division

Using objects, diagrams and pictorial representations to solve problems involving **both** grouping **and** sharing.
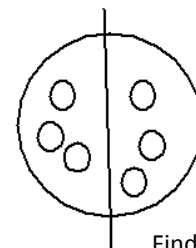
Pupils should :

- use lots of practical apparatus and picture representations

- Share objects into equal groups and count how many in each group

- Begin to count in multiples of 2s, 5s and 10s.

- Find **half** of a group of objects by sharing into 2 equal groups.

Sharing

counting in 2,5,10 etc.

Grouping

Find a half

Key Vocabulary: *share, share equally, one each, two each…, group, groups of, lots of,*

Key number skills needed for division at EYFS:

- Use own methods to work through a problems of sharing and grouping
- Through grouping and sharing small quantities, begin to understand, division,
- They make connections between number patterns, and counting in twos, fives and tens.
- Begin to understand halving using concrete objects and pictorial representations.

Division

Year 1   Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving **both** grouping **and** sharing.

Pupils should :

- use lots of practical apparatus, arrays and picture representations

- Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)

- Be able to count in multiples of 2s, 5s and 10s.

- Find **half** of a group of objects by sharing into 2 equal groups.

Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?
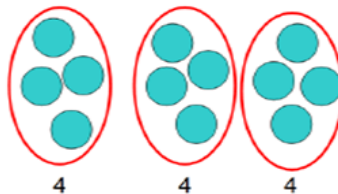
Can they work it out and give a division statement… ?

"18 shared between 6 people gives you 3 each."

Grouping:

How many groups of 4 can be made with 12 stars? = **3**

Sharing:

4   4   4

$12 \div 3 = 4$

12 shared between 3 is 4

Key Vocabulary: *share, share equally, one each, two each…, group, groups of, lots of, array*

Key number skills needed for division at Y1:
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.
- Begin to understand halving using concrete objects and pictorial representations.

Division

Year 2 Group <u>and</u> share, using the ÷ and = sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:
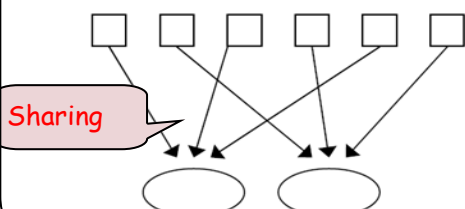
$12 ÷ 3 = 4$
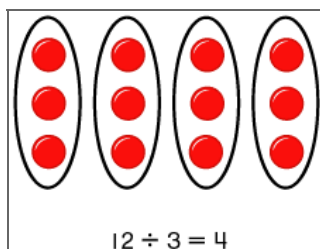
This represents **12 ÷ 3, posed as** how many groups of 3 are in 12?

Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

Know and understand sharing <u>and</u> grouping:

6 sweets shared between 2 people, how many do they each get?

Grouping

Sharing

There are 6 sweets, how many people can have 2 sweets each?

Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out 'how many groups of _ in _ ?'. Pupils could and using a bead string or practical apparatus to work out problems like 'A CD costs £3. How many CDs can I buy with £12?' **This is an important method to develop understanding of division as grouping.**

+3   +3   +3   +3

0 1 2 3 4 5 6 7 8 9 10 11 12
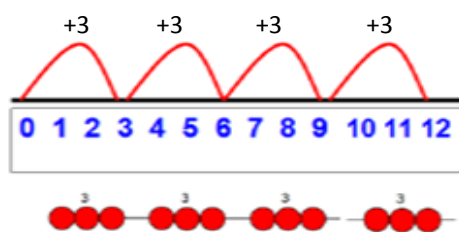
12 ÷ 3 = 4

Pose **12 ÷ 3** as '**How many groups of 3 are in 12?**'

Key Vocabulary: *share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over*

Key number skills needed for division at Y2:

- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
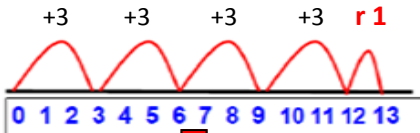
Division

Year 3 Divide 2-digit numbers by a single digit (where there is no remainder in the final answer)

$7 \div 2 = 3$ R 1
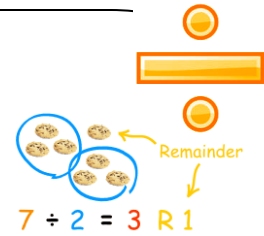
Remainder

Grouping on a number line:

$13 \div 3 = 4$ r 1

+3 +3 +3 +3 **r 1**

0 1 2 3 4 5 6 7 8 9 10 11 12 13

STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of **remainders,** as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for 'carrying' remainders across within the short division method.

Short division: Limit numbers to NO remainders in the answer OR carried (each digit must be a multiple of the divisor).

$$3 \overline{)9\ 6}$$
$$3\ 2$$

STEP 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by introducing the layout of short division by comparing it to an array.

Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:

- *How many 3's in 9? = 3, and record it above the **9 tens**.*
- *How many 3's in 6? = 2, and record it above the **6 units**.*

8
7   56

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Short division: Limit numbers to NO remainders in the final answer, but with remainders occurring within the

$$4 \overline{)7\ ^3 2}$$
$$1\ 8$$

STEP 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 96÷4), and be taught to 'carry' the remainder onto the next digit.

Step 3 Only taught when pupils can calculate **'remainders'**.

Key Vocabulary: *share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over,* ***inverse, short division, 'carry', remainder, multiple***

Key number skills needed for division at Y3:

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Division

Year 4 Divide up to 3-digit numbers by a single digit

(without remainders initially)

Continue to develop short division:

> Short division should only be taught once children have secured the skill of calculating 'remainders'.

$$4 \overline{)7\,{}^3 2} = 18$$

STEP 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (**those that do not result in a final remainder** —see steps in Y3), but must understand how to calculate remainders, using this to 'carry' remainders within the calculation process (see example).

$$4 \overline{)8\,7\,{}^3 2} = 218$$

STEP 2: Pupils move onto dividing numbers with up to **3-digits** by a single digit, however problems and calculations provided should **not result in a final answer with remainder** at this stage. Children who exceed this expectation may progress to Y5 level.

$$5 \overline{)1\,{}^1 8\,{}^3 5} = 037$$

When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1) over to the next digit as a remainder.

> Include money and measure contexts when confident.

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Key Vocabulary: *share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple,* **divisible by, factor**

Key number skills needed for division at Y4:

- **Recall multiplication and division facts for all numbers up to 12 x 12.**
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Division

Year 4+ Divide up to 4 digits by a single digit, including those with remainders.

Short division, including remainder answers:

Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it,** ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.

The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem involved.

See Y6 for how to continue the short division to give a decimal answer for children who are confident.

Include money and measure contexts.

Approximate, Calculate, Check.

If children are confident and accurate:

- Introduce **long division** for pupils who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Year 6 expectation—see Y6.

Key Vocabulary: *share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse,* **quotient, prime number, prime factors, composite number (non-prime)**

Key number skills needed for division at Y4+:
- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r2 = 24$\frac{1}{2}$ = 24.5 ≈ 25).
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.